Experimental Simulations of Intense Beam Propagation Over Large Distances in a Compact Linear Paul Trap*

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2005 APS-DPP
Denver, CO
October 26th, 2005

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*This work is supported by the U.S. Department of Energy.

PTSX Simulates Nonlinear Beam Dynamics in Magnetic Alternating-Gradient Systems

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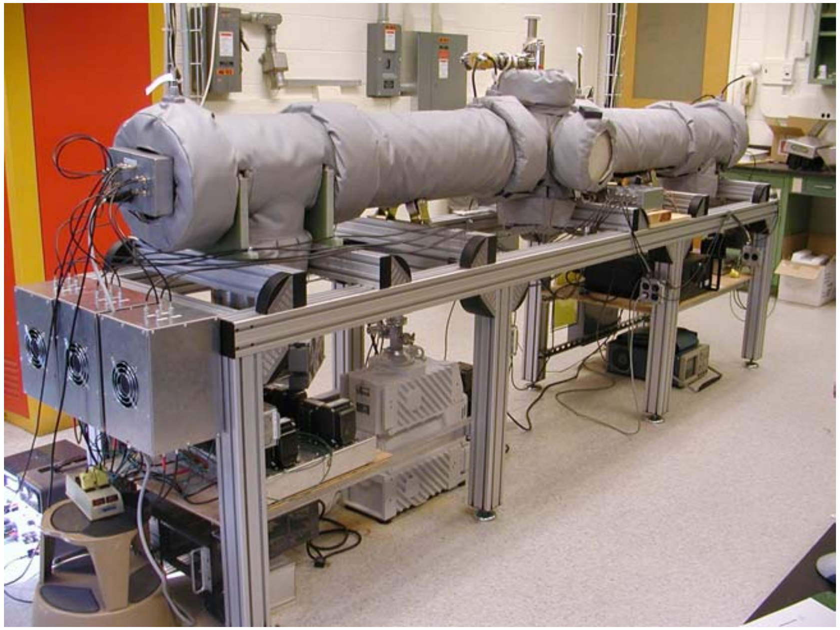
Purpose: Simulate the nonlinear transverse dynamics of intense beam propagation over large distances through magnetic alternating-gradient transport systems in *a compact experiment*.

Applications: Accelerator systems for high energy and nuclear physics applications, high energy density physics, heavy ion fusion, spallation neutron sources, and nuclear waste transmutation.



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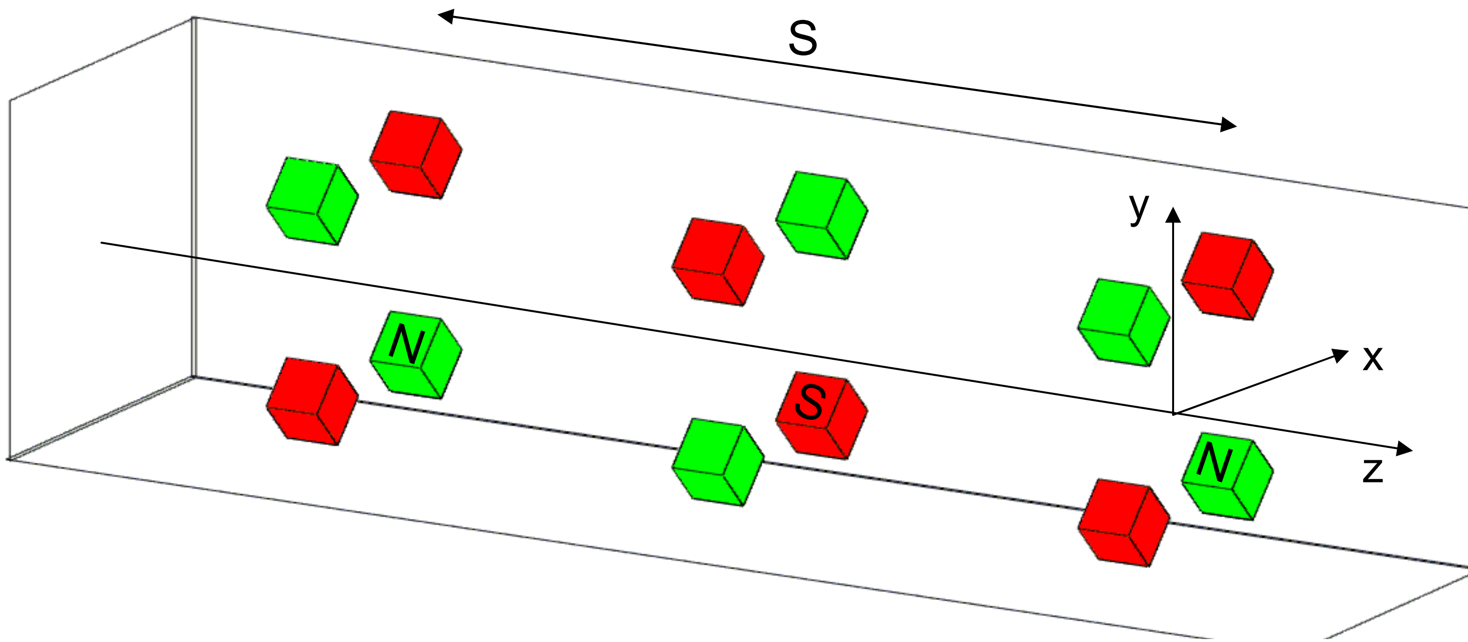
Scientific Motivation

- Beam mismatch and envelope instabilities
- Collective wave excitations
- Chaotic particle dynamics and production of halo particles
- Mechanisms for emittance growth
- Effects of distribution function on stability properties
- Compression techniques

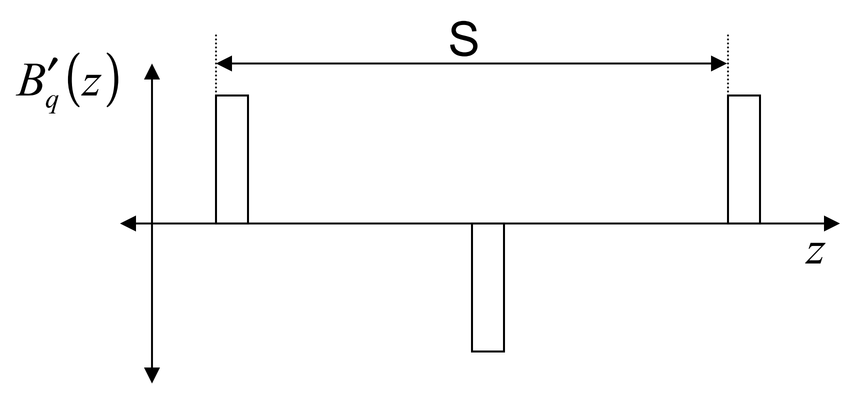
Magnetic Alternating-Gradient Transport Systems



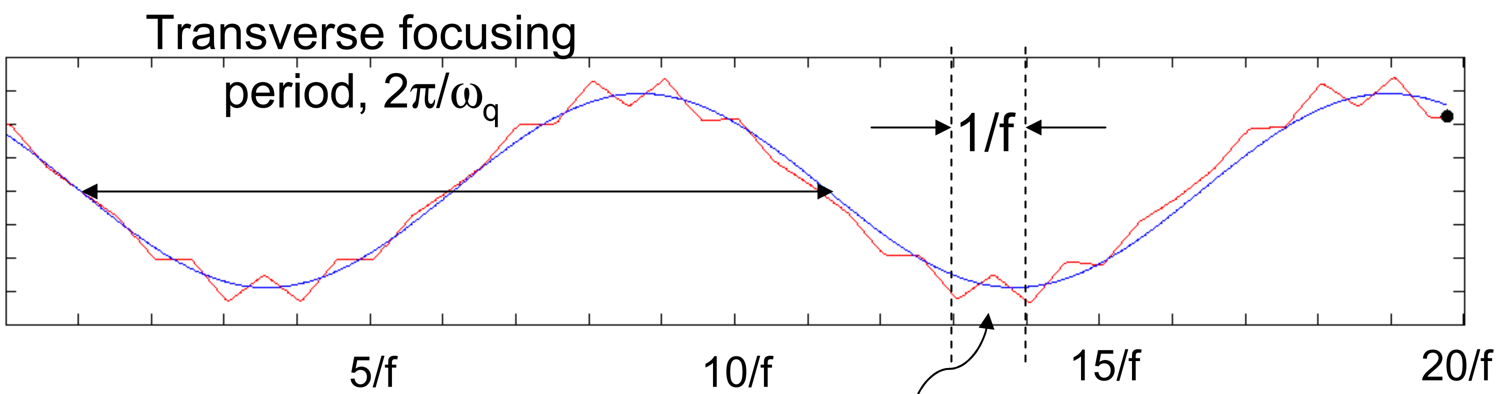
$$\boldsymbol{B}_q^{foc}(\boldsymbol{x}) = B_q'(z)\left(y\hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y\right)$$

$$\boldsymbol{F}_{foc}(\boldsymbol{x}) = -\kappa_q(z)\left(x\hat{\boldsymbol{e}}_x - y\hat{\boldsymbol{e}}_y\right)$$

$$\kappa_q(z) \equiv \frac{ZeB_q'(z)}{\gamma m\beta c^2}$$



Transverse Focusing Frequency, Vacuum Phase Advance, and Normalized Intensity Parameter Characterize the System



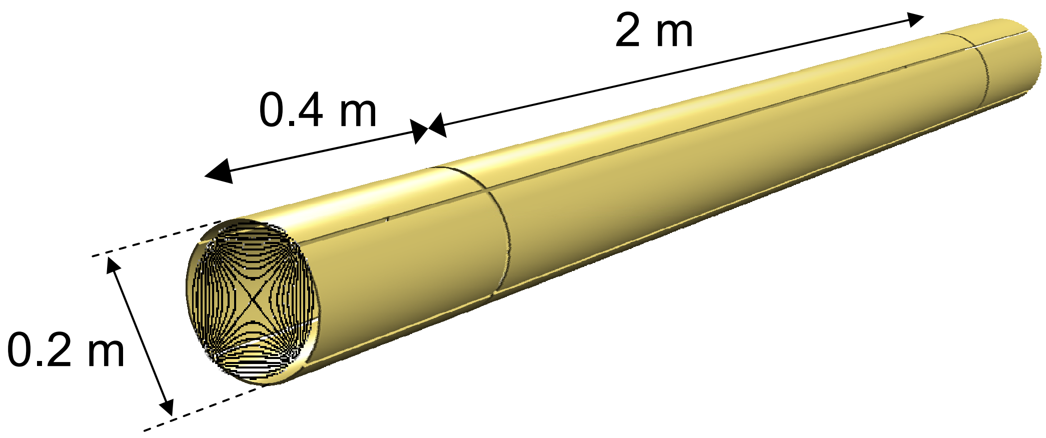
The smooth trajectory's vacuum phase advance, σ_v, is 35 degrees in this example.

To avoid instabilities, $\sigma_v = \dfrac{\omega_q}{f} < \sigma_{v\max}$

Normalized intensity parameter $s \equiv \dfrac{\omega_p^2}{2\omega_q^2} < 1$ to confine the space-charge.

s ~ 0.2 for Spallation Neutron Source.

PTSX Configuration – A Cylindrical Paul Trap



$$e\phi_{ap}(x,y,t) = \frac{1}{2}\kappa_q'(t)(x^2 - y^2)$$

$$\kappa_q'(t) = \frac{8eV_0(t)}{m\pi r_w^2}$$

$$\omega_q = \frac{8eV_{0\max}}{m\pi r_w^2 f}\xi$$

Plasma column length	2 m	Maximum wall voltage	~ 400 V
Wall electrode radius	10 cm	End electrode voltage	< 150 V
Plasma column radius	~ 1 cm	Voltage oscillation frequency	< 100 kHz
Cesium ion mass	133 amu		

Transverse Dynamics are the Same Including Self-Field Effects

If…

- Long coasting beams
- Beam radius << lattice period
- Motion in beam frame is nonrelativistic

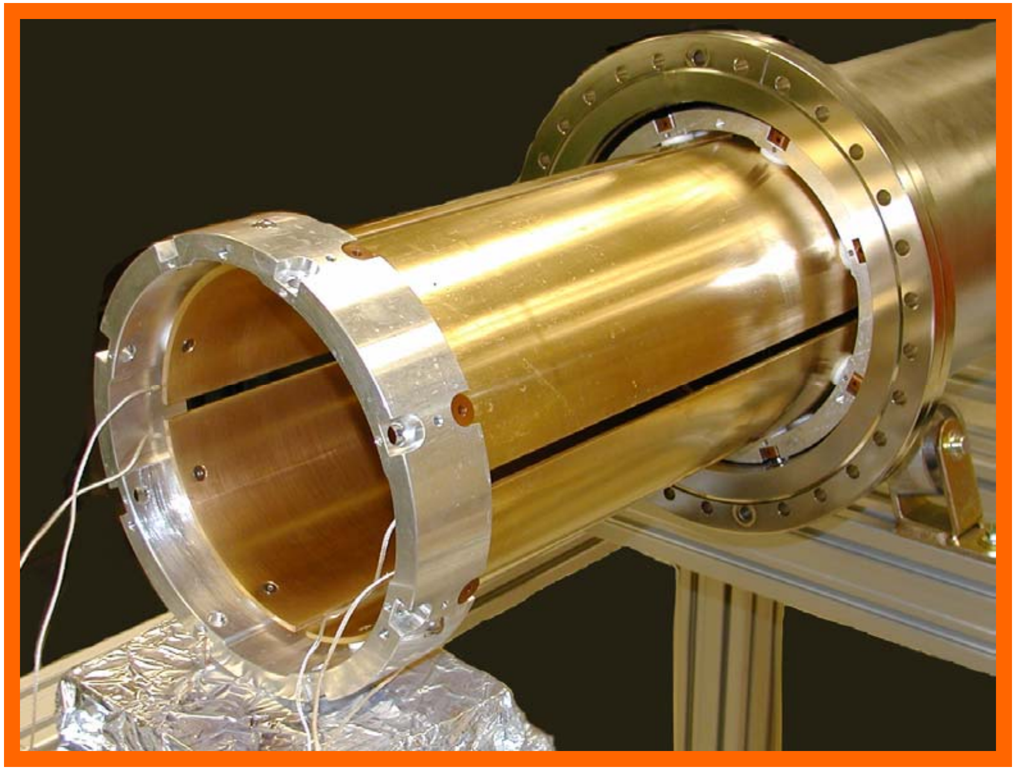
Then, when in the beam frame, both systems have…

- Quadrupolar external forces
- Self-forces governed by a Poisson equation
- Distributions evolve according to Vlasov equation

→ Ions in PTSX have the same transverse equations of motion as ions in an alternating-gradient system ***in the beam frame***.

Electrodes, Ion Source, and Collector

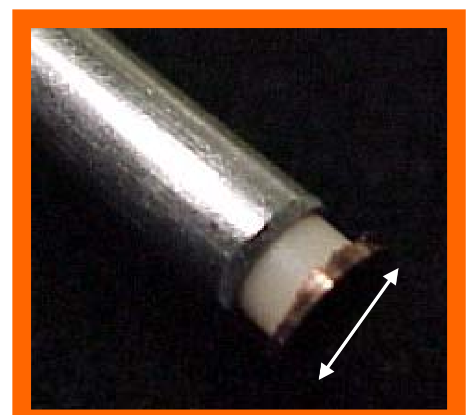
Broad flexibility in applying V(t) to electrodes with arbitrary function generator.



Increasing source current creates plasmas with ***s* up to 0.8.**

1.25 in

Large dynamic range using sensitive electrometer.



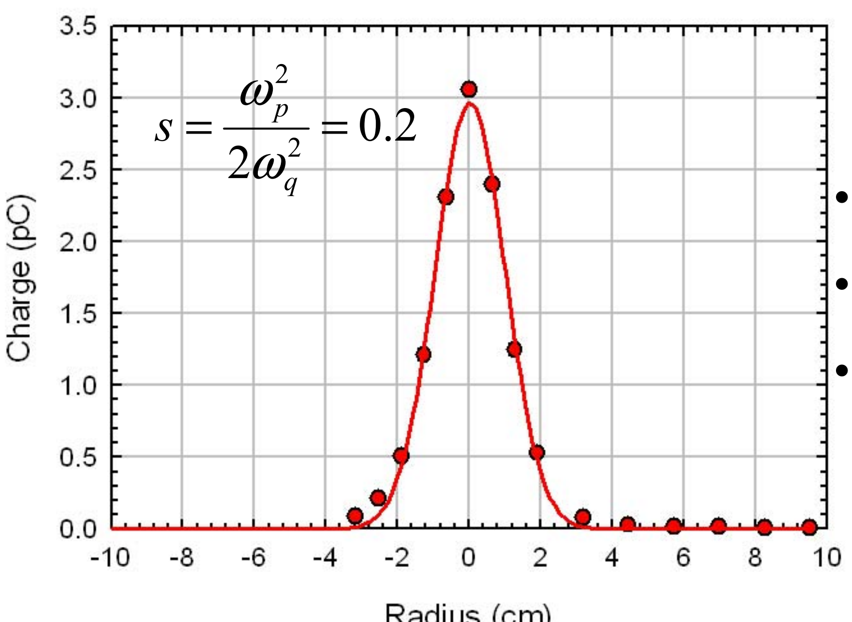
5 mm

Radial Profiles of Trapped Plasmas are Approximately Gaussian – Consistent with Thermal Equilibrium

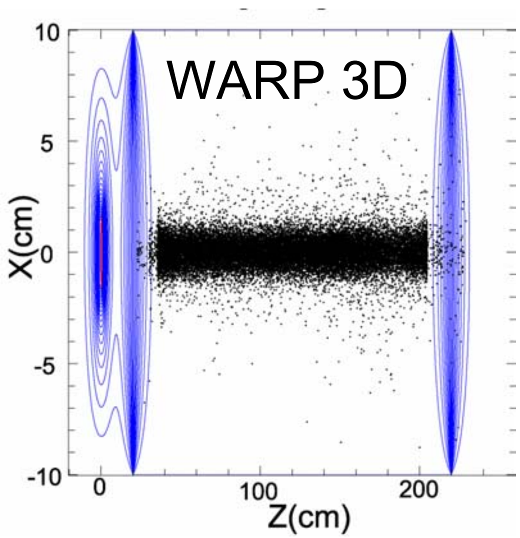
$$n(r) = n(0)\exp\left[-\frac{m\omega_q^2 r^2 + 2q\phi^s(r)}{2kT}\right]$$

- $V_{0\,max}$ = 235 V
- f = 75 kHz
- t_{hold} = 1 ms
- σ_v = 49°
- $\omega_q = 6.5 \times 10^4\ s^{-1}$

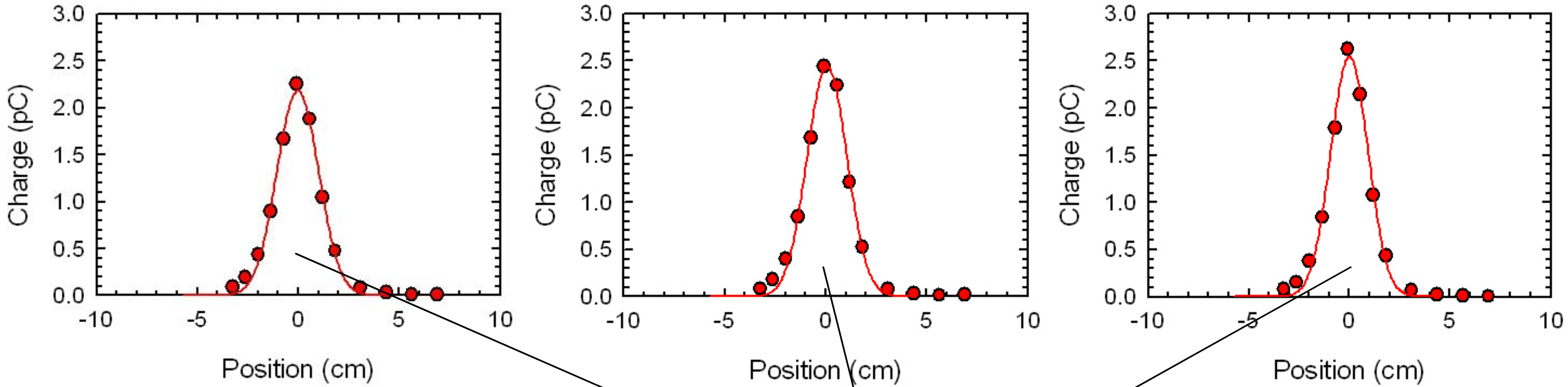
$$n(r) = \frac{Q(r)}{e\pi r_{aperture}^2 l_{plasma}}$$

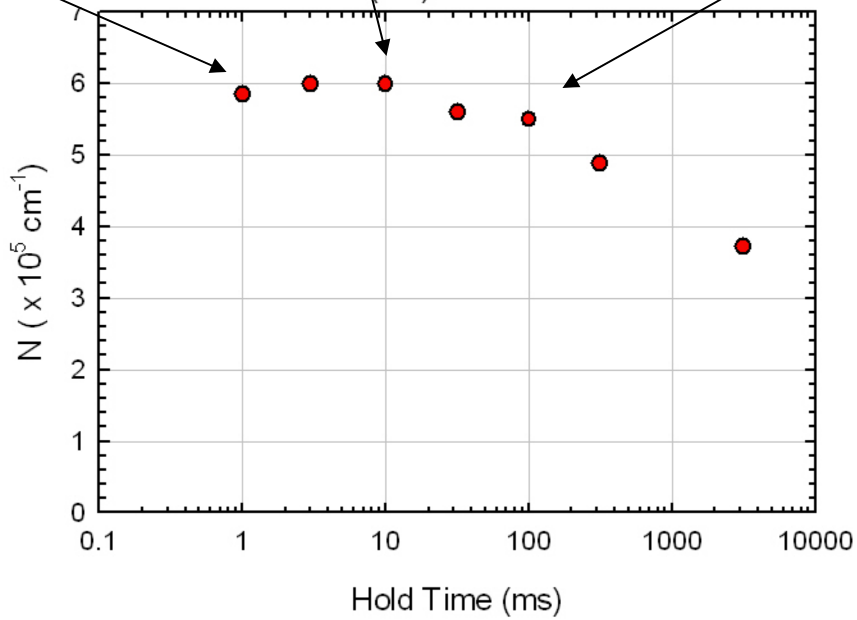


- $n(0) = 1.4\times10^5\ cm^{-3}$
- R = 1.4 cm
- s = 0.2



PTSX Simulates Equivalent Propagation Distances of 7.5 km

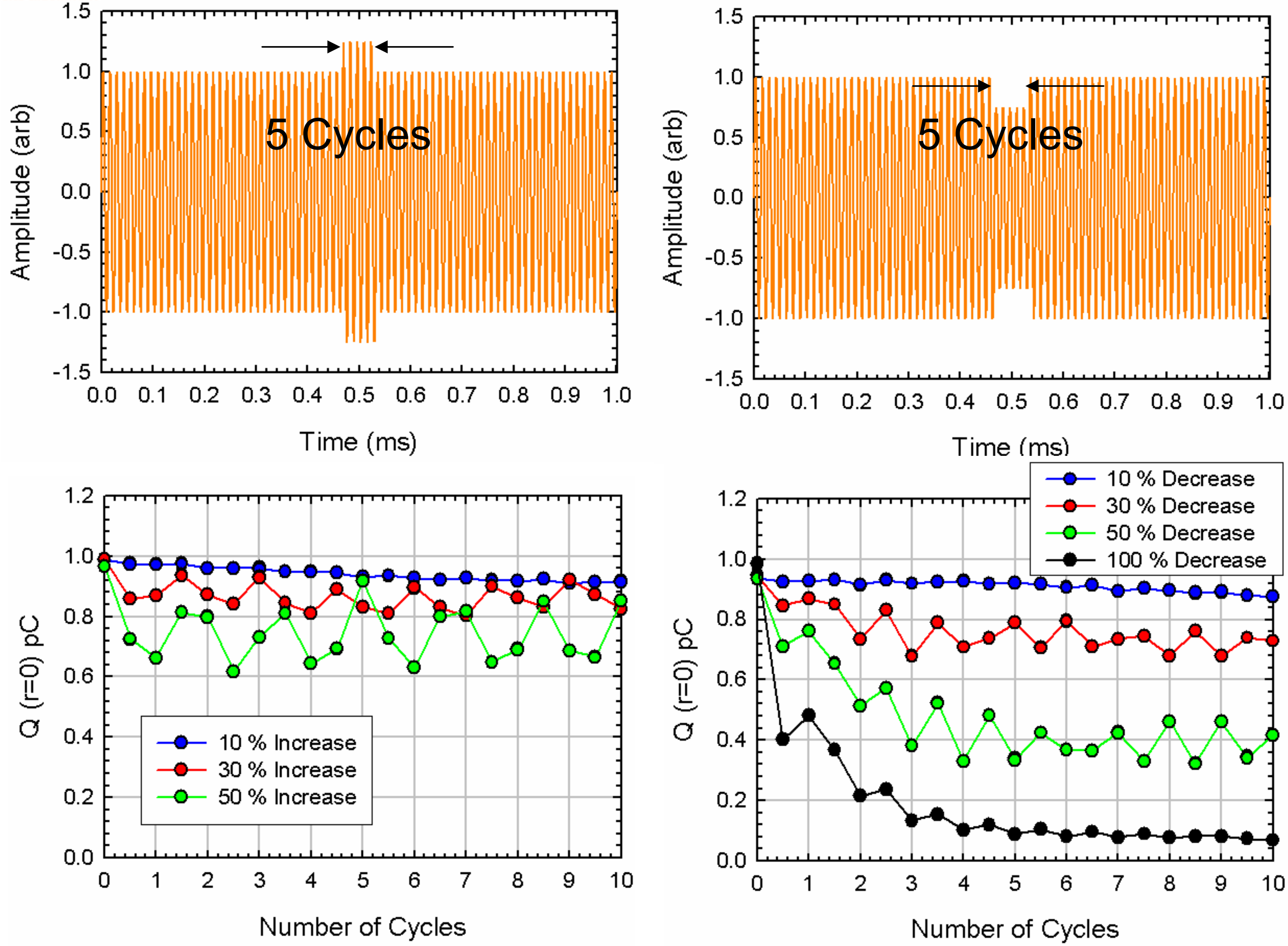




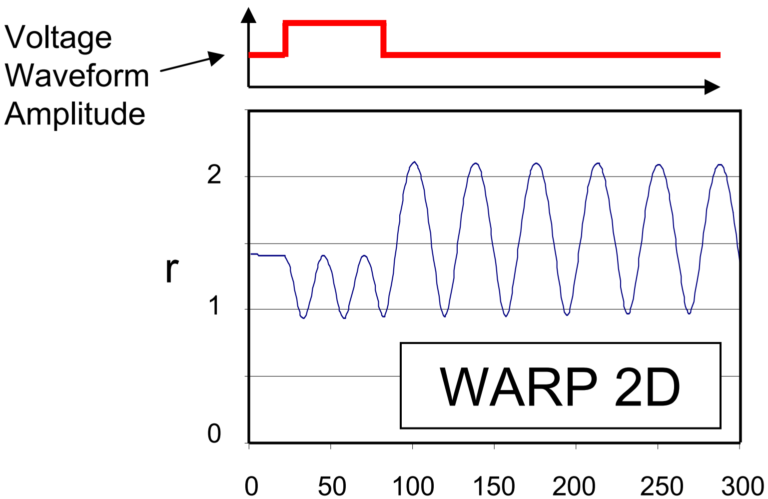
- At f = 75 kHz, a lifetime of 100 ms corresponds to **7,500 lattice periods**.
- If lattice period is 1 m, the PTSX simulation experiment would correspond to a **7.5 km beamline**.

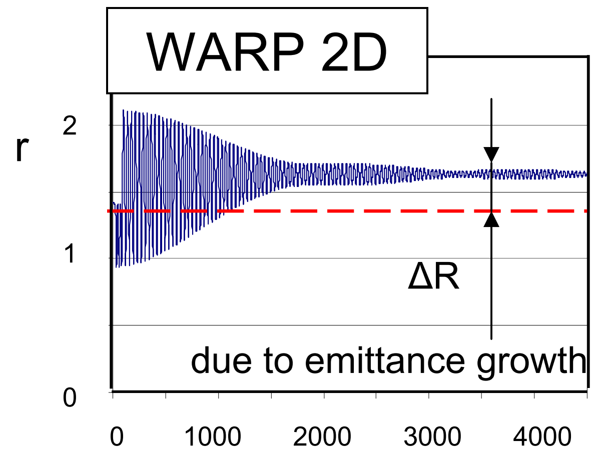
- $s = \omega_p^2 / 2\omega_q^2 = 0.18$.
- $V_{0\,max}$ = 235 V
  f = 75 kHz
  $\sigma_v = 49^o$

Temporarily Changing the Amplitude Causes the Plasma Envelope to Oscillate



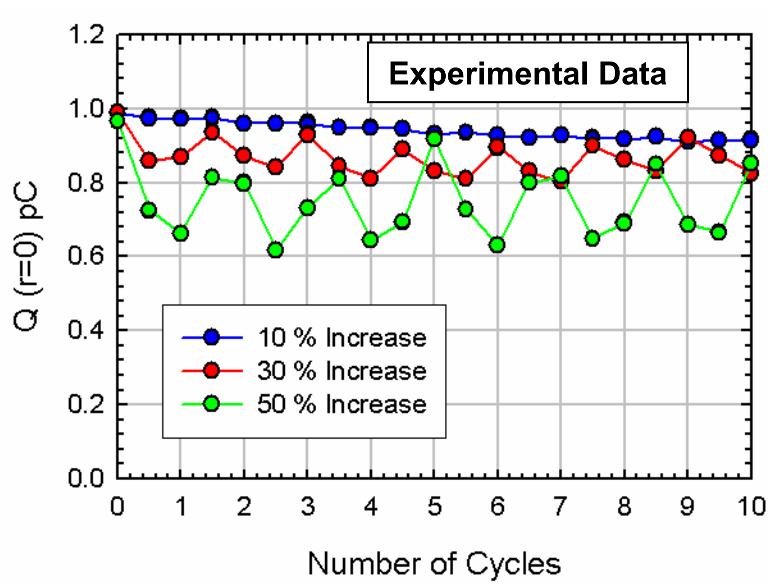
The Mismatch When the Amplitude is Restored Depends on the Frequency of the Envelope Oscillation

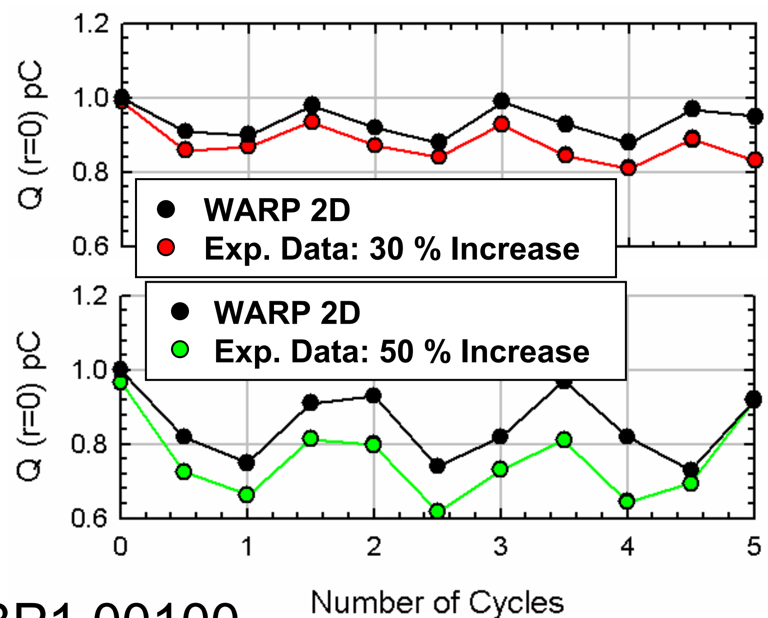




Force Balance

$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$





M. Dorf BP1.00100

Comparison of Instantaneous Changes in Waveform Amplitude and Frequency

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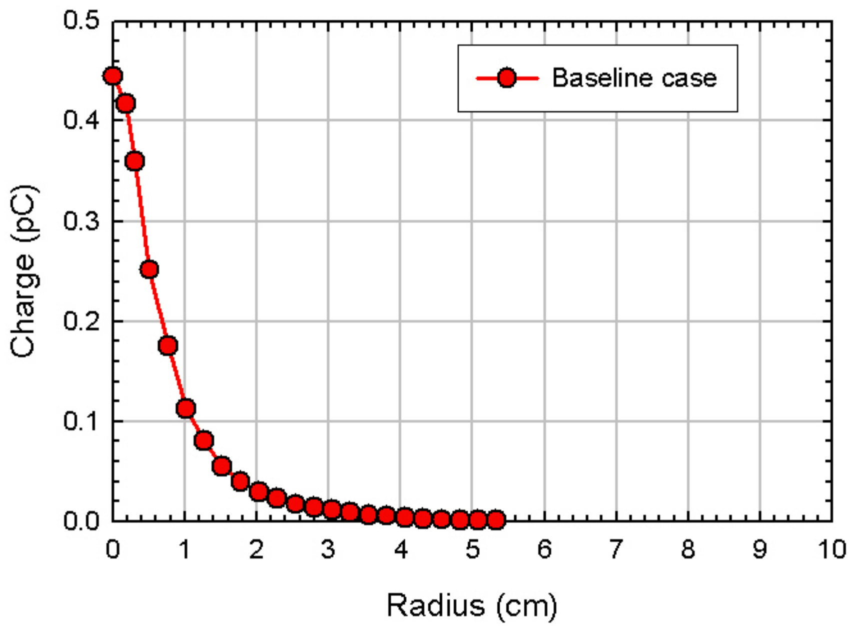


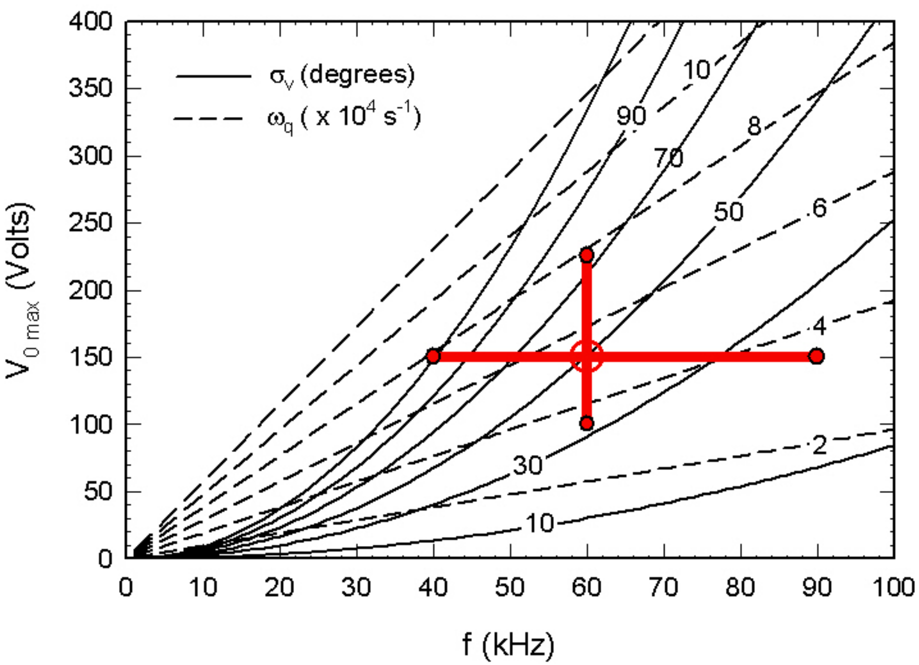
Baseline case:

V_1 = 150 V	$\omega_q = 52{,}200\ s^{-1}$	s = 0.2	t = 1 ms
f = 60 kHz	$\sigma_v = 49^o$	kT ~ 0.7 eV	

Comparison of Instantaneous Changes in Waveform Amplitude and Frequency

$$\omega_q = \frac{8eV_{0\max}}{m\pi r_w^2 f}\xi$$

$$\sigma_v = \frac{\omega_q}{f}$$



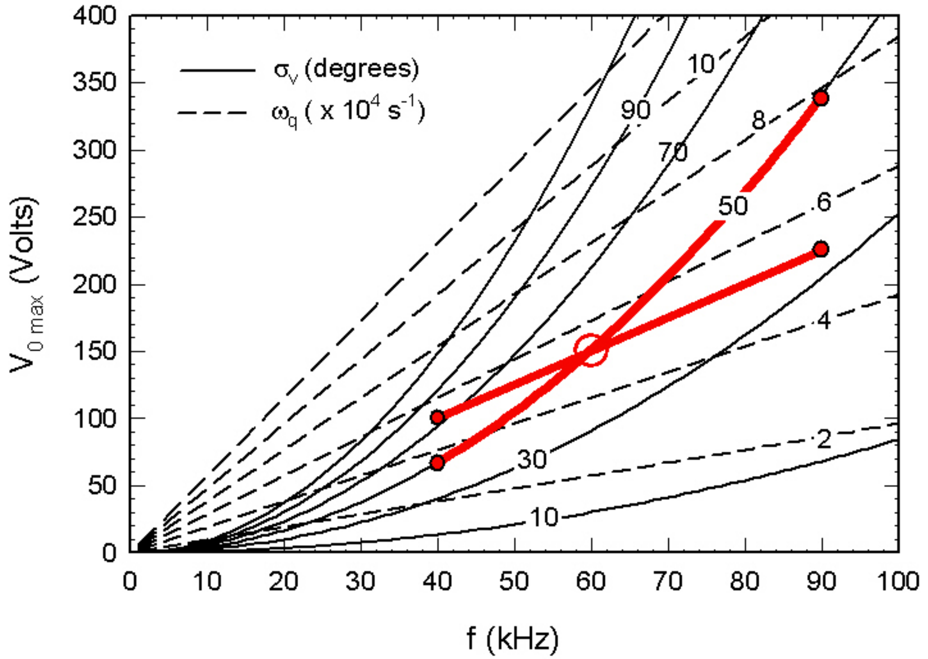
Constant voltage $\omega_q \rightarrow \omega_q/1.5,\ \omega_q \rightarrow 1.5\ \omega_q$

Constant frequency $\omega_q \rightarrow \omega_q/1.5,\ \omega_q \rightarrow 1.5\ \omega_q$

Comparison of Instantaneous Changes in Waveform Amplitude and Frequency

$$\omega_q = \frac{8eV_{0\,max}}{m\pi r_w^2 f}\xi$$

$$\sigma_v = \frac{\omega_q}{f}$$



Constant transverse focusing frequency

Constant vacuum phase advance $\omega_q \rightarrow \omega_q/1.5$, $\omega_q \rightarrow 1.5\,\omega_q$

Comparison of Instantaneous Changes in Waveform Amplitude and Frequency

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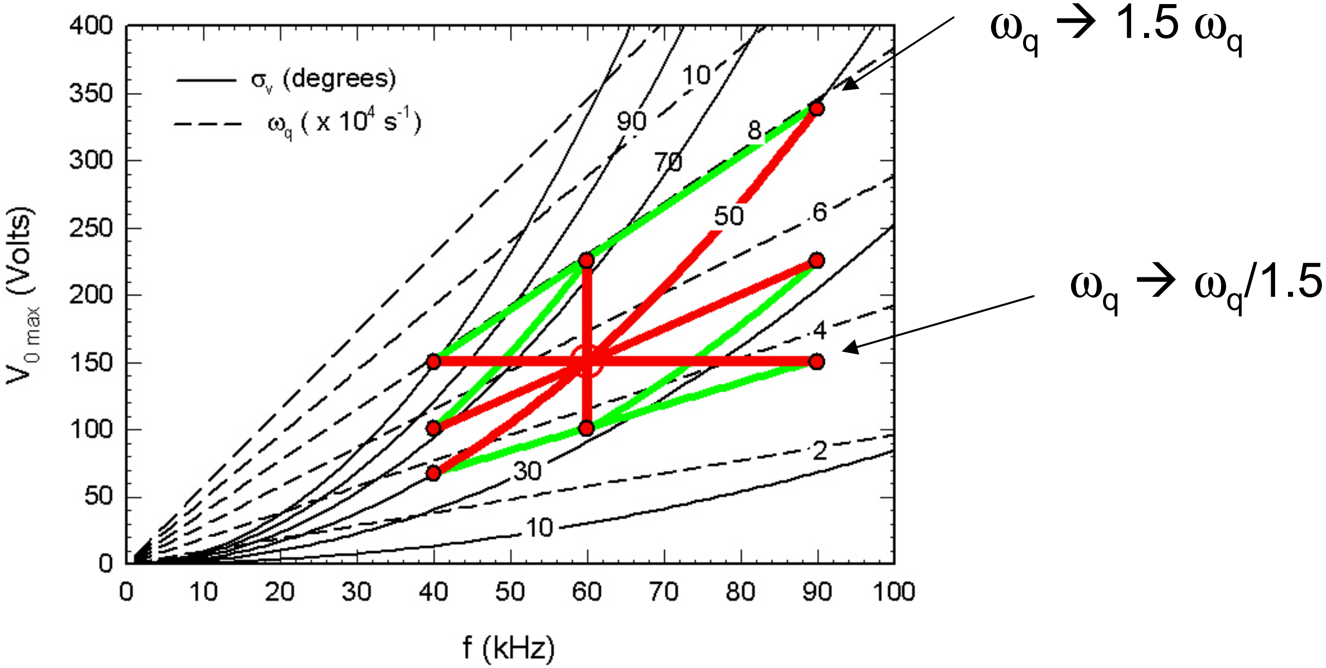
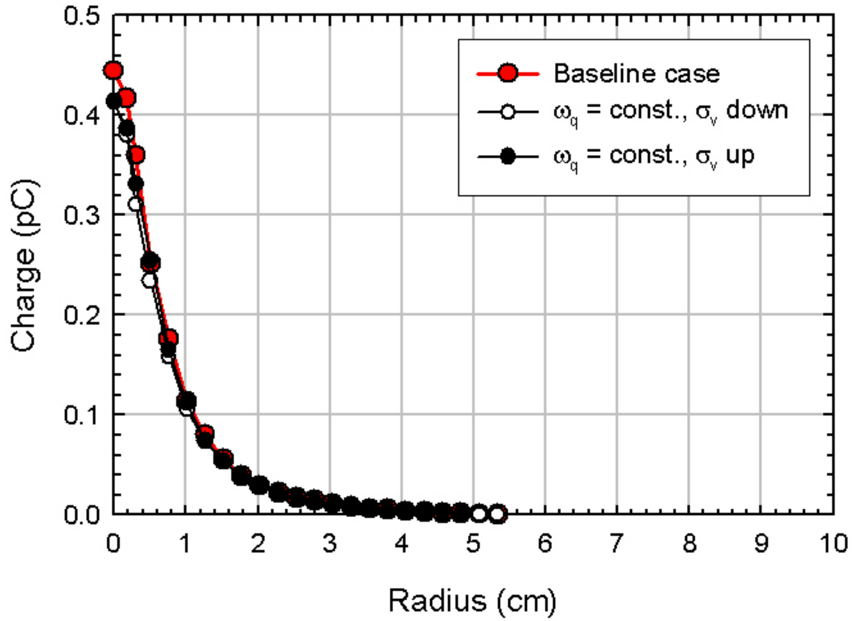
$$\omega_q = \frac{8eV_{0\,max}}{m\pi r_w^2 f}\xi$$

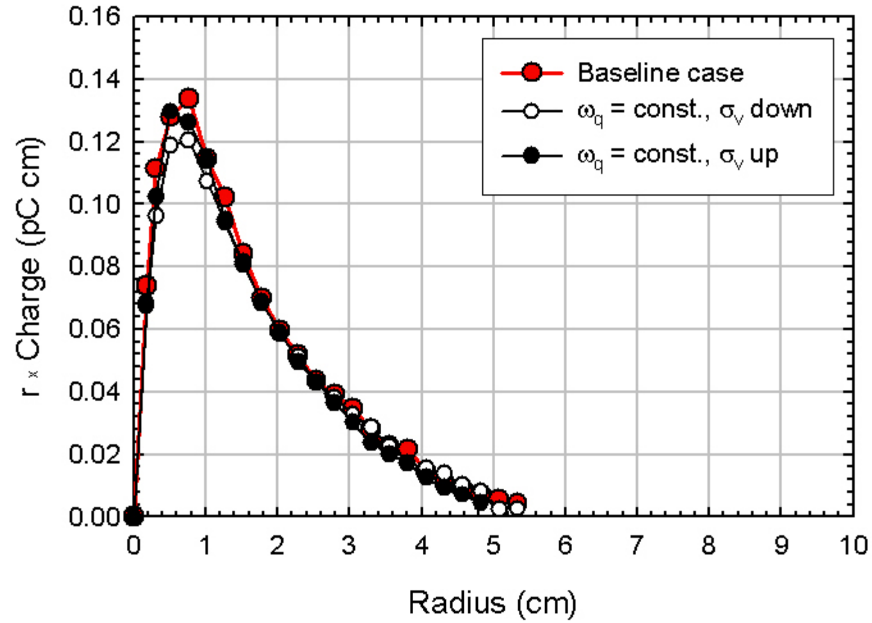
$$\sigma_v = \frac{\omega_q}{f}$$



N ~ constant in following data.

Changing the Voltage and Frequency Does Not Affect the Transvese Profile When the Transverse Focusing Frequency is Fixed



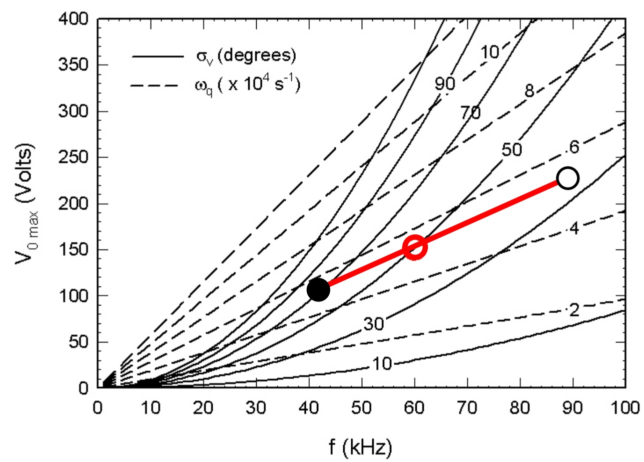


s = 0.2

kT ~ 0.7 eV

N ~ constant

$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$



$\sigma_v = 33^o$

$\sigma_v = 50^o$

$\sigma_v = 75^o$

Decreasing Transverse Focusing Frequency Preserves Normalized Intensity but Increases Emittance

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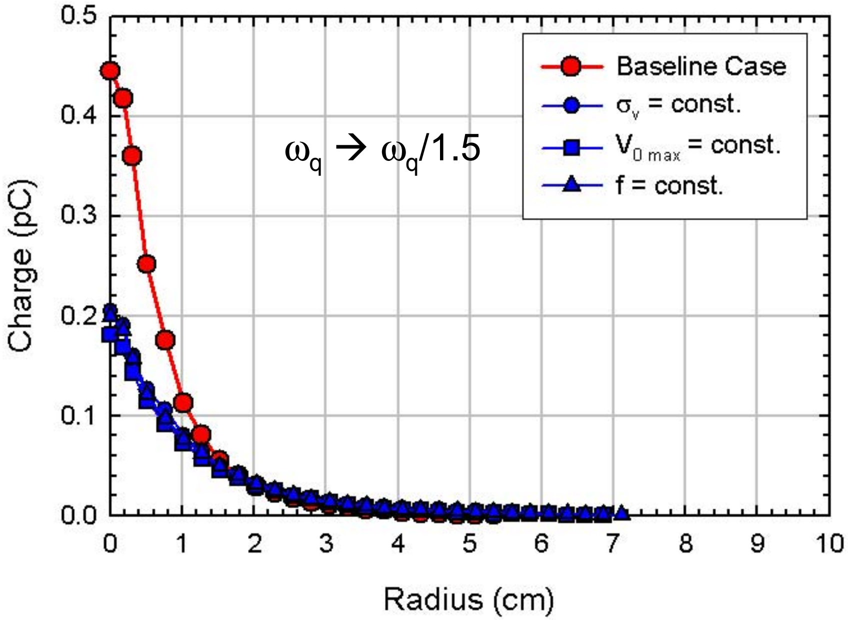
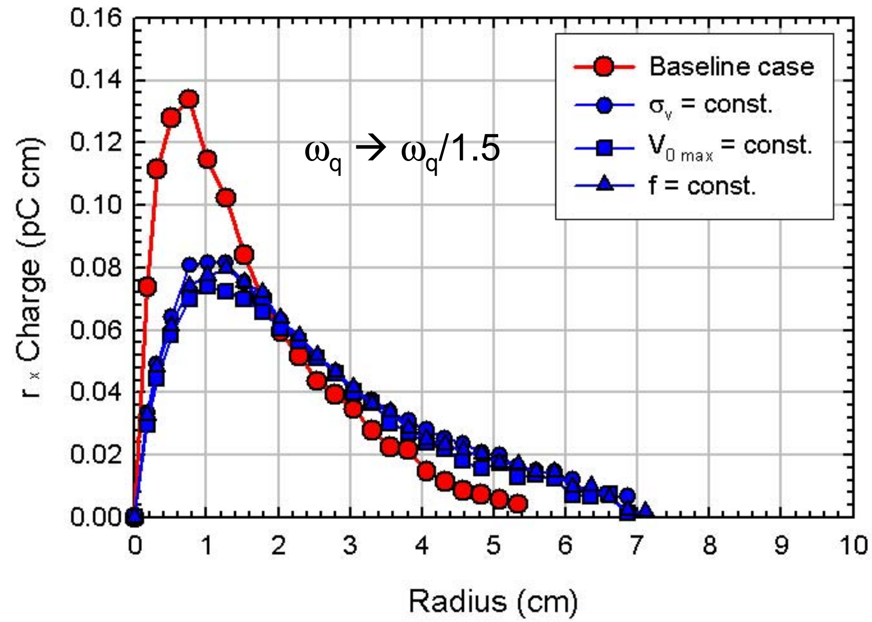
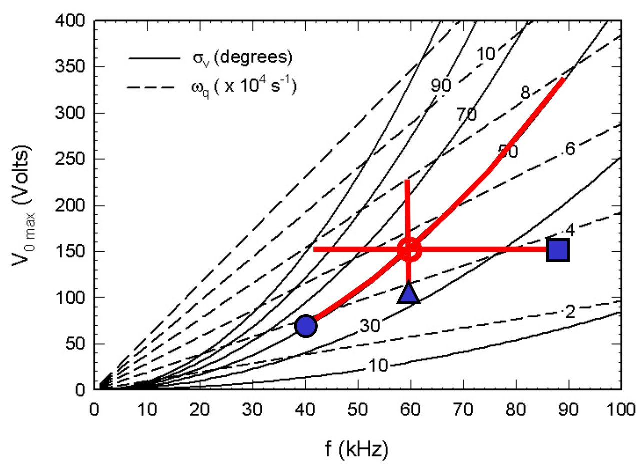




- ■ s = 0.2
- ▲ kT ~ 0.7 eV
- ● N ~ constant

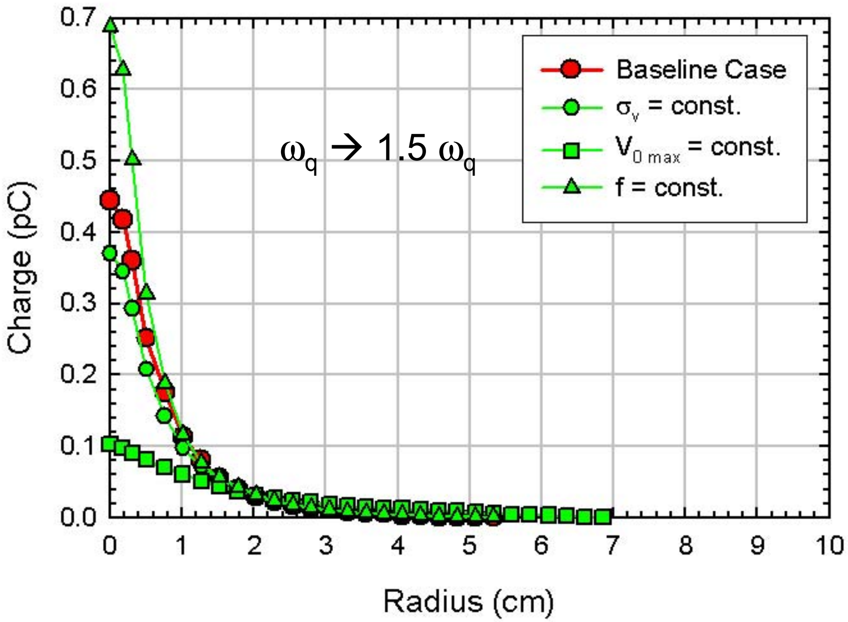
$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$

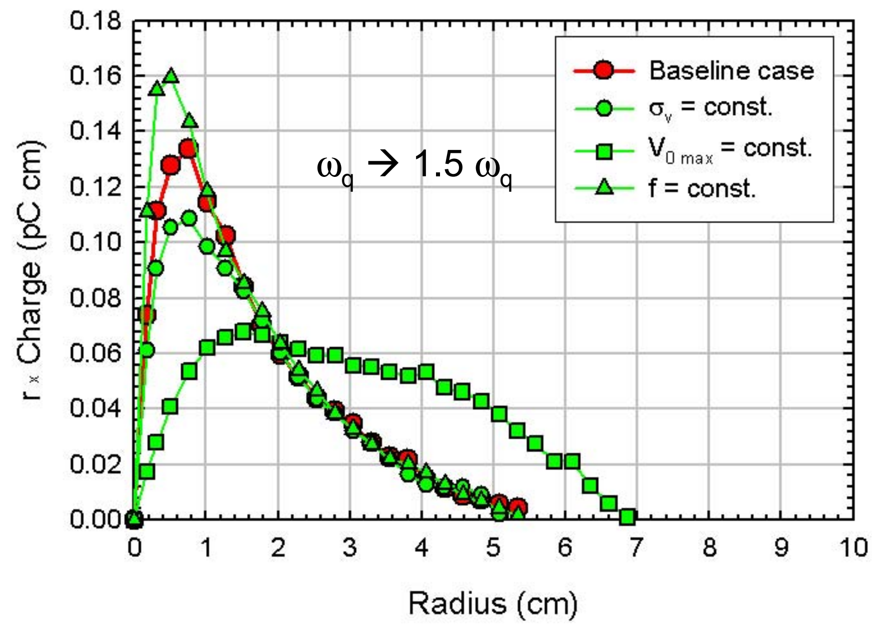


- ■ $\sigma_v = 22^o$
- ▲ $\sigma_v = 33^o$
- ● $\sigma_v = 50^o$

Emittance increases by 45%

Increasing Phase Advance Degrades Transverse Confinement

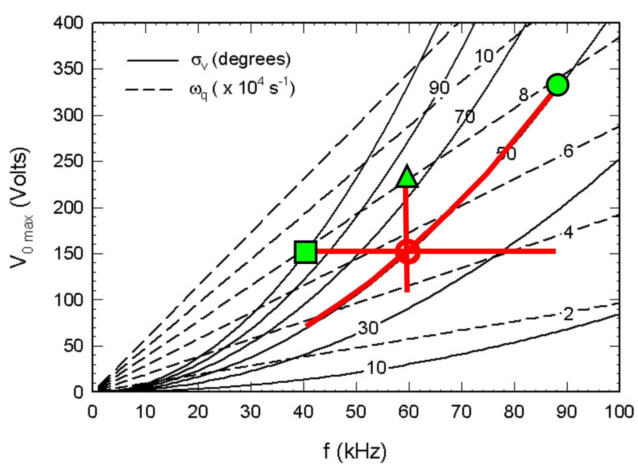




N ~ constant

- s = 0.08, kT = 1.6 eV (circle)
- s = 0.14, kT = 1.4 eV (triangle)
- s = 0.02, kT = 4.7 eV (square)

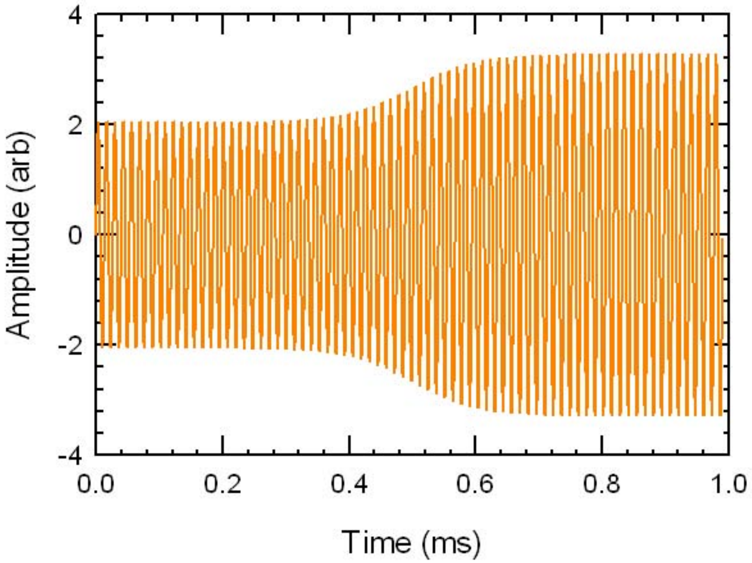
$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$



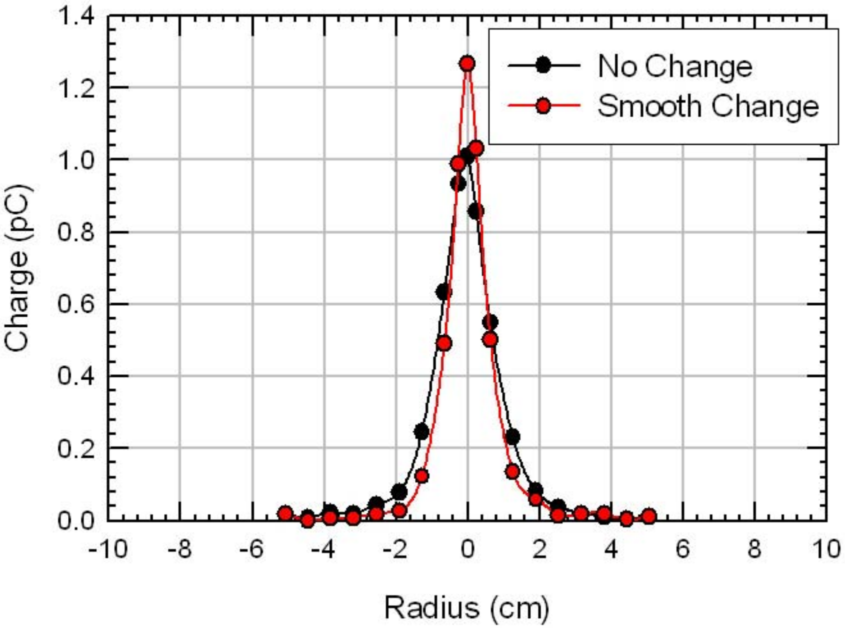
Emittance increases

$\sigma_v = 50^o$ (circle)	60%
$\sigma_v = 75^o$ (triangle)	40%
$\sigma_v = 112^o$ (square)	450%

A Smooth Change in Lattice Strength Compresses the Plasma: a First Look

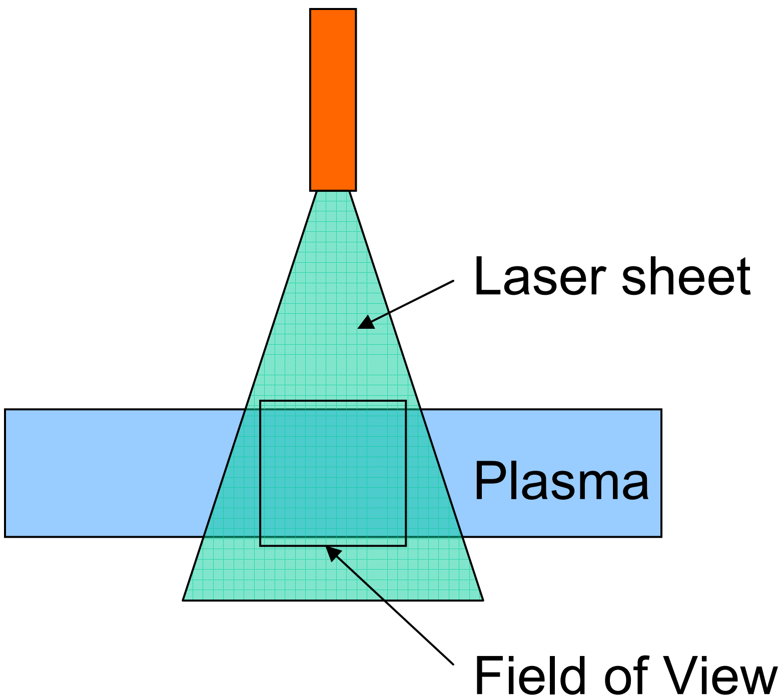


A hyperbolic tangent transition from one amplitude to another at fixed frequency.



LIF System is Being Installed

- Nondestructive
- Image entire transverse profile at once
- Time resolution
- Velocity measurement



M. Chung BP1.00085

PTSX Simulates the Transverse Dynamics of Intense Beam Propagation Over Large Distances Through Magnetic Alternating-Gradient Transport Systems

- PTSX is a compact and flexible laboratory experiment.
- PTSX can trap plasmas with normalized intensity *s* up to 0.8.
- Confinement times can correspond to up to 7,500 lattice periods.
- Instantaneous lattice changes are detrimental to transverse confinement, leading to mismatch, envelope oscillations, and subsequent emittance growth (unless transverse focusing frequency is fixed.) Large vacuum phase advance σ_v degrades transverse confinement.
- Study of smooth lattice changes is planned.